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The big bounce in rainbow universe

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ABSTRACT

The usual Einstein's equations is modified as a one parameter family of equations in the framework of rainbow gravity. In this Letter we derive the modified Friedmann–Robertson–Walker (FRW) equations when the cosmological evolution of radiation particles is taken into account. In particular, given some specific dispersion relations, the big bounce solutions to the modified FRW equations can be derived. Notably, to obtain a well defined rainbow metric at the moment of the big bounce, we find it seems necessary to introduce a cosmological constant which depends on the energy of probes as well, implying that a universe with a positive cosmological constant more likely undergoes a big bounce at least at this phenomenological level.

1. Introduction

Before a complete and fundamental quantum theory of gravity can be established, it has been suggested that a semi-classical or phenomenological theory of quantum gravity may play a crucial role in disclosing the nature of quantum gravity effects, in particular its possible impact on the very early universe and extremely high energy physics [1–10]. Recently such a semi-classical formalism named as rainbow gravity has been proposed by Magueijo and Smolin, which can also be viewed as an extension of doubly special relativity in curved spacetime [11]. In this formalism, one key ingredient is that there is no single fixed spacetime background when the quantum gravity effects of moving probes on geometry is taken into account. Instead, it is replaced by a one parameter family of metrics which depends on the energy of these probes, forming a "rainbow" metric. More explicitly, suppose the modified dispersion relation in doubly special relativity has a general form

$$\varepsilon^2 f^2(l_p\varepsilon) - p^2 g^2(l_p\varepsilon) = m_0^2, \tag{1}$$

where two general rainbow functions $f^2(\varepsilon)$ and $g^2(\varepsilon)$ depend on the energy ε of probes and may be expanded in the order of Planck length $l_p = \sqrt{8\pi G} \sim 1/M_p$. Obviously one requires that $f^2(\varepsilon), g^2(\varepsilon) \sim 1$ as $\varepsilon/M_p \ll 1$. Correspondingly, it is conjectured that the usual flat metric should be replaced by the rainbow metric defined as

$$ds^2 = -\frac{1}{f^2(\varepsilon)}dt^2 + \frac{1}{g^2(\varepsilon)}dx^2, \tag{2}$$

such that the contraction between infinitesimal displacement and momenta is a linear invariant [11,12],

$$dx^\mu p_\mu = dt\,\varepsilon + dx^j p_j. \tag{3}$$

The flat rainbow metric (2) indicates that the geometry of spacetime depends on the energy of a particle moving on it. That is to say, even in the absence of gravity a moving particle with energy ε would probe a geometry which is described by an energy-dependent set of orthonormal frame fields

$$e_0 = f^{-1}(l_p\varepsilon)\tilde{e}_0, \qquad e_i = g^{-1}(l_p\varepsilon)\tilde{e}_i, \tag{4}$$

where the tilde quantities are the energy independent frame fields probed by low energy particles. In this manner the metric can be written as

$$g(\varepsilon) = \eta^{ab} e_a(\varepsilon) \otimes e_b(\varepsilon), \tag{5}$$

which can be viewed as a one family of flat metrics, characterized by ε. This strategy overcomes the difficulty of defining the position space conjugate to the momentum space arising in *DSR* where the Lorentz symmetry is accomplished by non-linear transformations in momentum space (for recent progress and discussion on this issue we refer to [13]).

Rainbow metric formalism can be pushed forward when the gravity is taken into account. First of all, a deformed equivalence principle of general relativity is proposed in [11], requiring that the free falling observers who make measurements with energy ε

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will observe the same laws of physics as in doubly special relativity. As a consequence, the background spacetime is described by a general rainbow metric $g_{\mu\nu}(\varepsilon)$. Then, through the standard process the corresponding one-parameter family of connection $\nabla(\varepsilon)_\mu$ which is compatible with the rainbow metric $g_{\mu\nu}(\varepsilon)$ and the curvature tensor $R(\varepsilon)^{\rho}_{\mu\nu\lambda}$ can be constructed, leading to a family of modified Einstein's equations

$$G_{\mu\nu}(\varepsilon) = 8\pi G(\varepsilon) T_{\mu\nu}(\varepsilon) + g_{\mu\nu}(\varepsilon)\Lambda(\varepsilon), \tag{6}$$

where the Newton's constant $G(\varepsilon)$ and the cosmological constant $\Lambda(\varepsilon)$ are expected to vary with the energy ε as well from a renormalization group point of view.

Rainbow gravity formalism has received a lot of attention recently and other stimulating work on this formalism can be found, for instance, in [14–25]. In particular, in [21] one of the authors generalized the modified Friedmann–Robertson–Walker (FRW) equations originally presented in [11] by considering the cosmological evolution of probes and derived solutions in which the spacetime curvature has an upper bound such that the cosmological singularity is absent. But in [21] the big bounce solution to the modified FRW equations is not available, which greatly depends on what kind of modified dispersion relations we would apply.

Another motivation of looking for non-singular bouncing solutions in rainbow gravity formalism comes from recent progress in cosmology. It suggests that bouncing universes could play a similar role as inflationary scenario in solving the well-known problems in standard Big-Bang cosmology. Nowadays there are a lot of cosmological models with a bounce solution such as the pre Big-Bang scenario [26] and cyclic/Ekpyrotic universe [27,28], as well as superstring cosmology [29–33], brane cosmology [34,35], loop quantum cosmology [36–38] and quintom models [39,40]. For more details on bouncing universe we refer to a recent review [41] and references therein. Since rainbow gravity formalism is proposed as a semi-classical theory in which the quantum effects of gravity is taken into account, we would like to ask if it is possible to obtain such big bounce solutions in this framework. Bearing this question in mind, in this Letter we intend to further investigate the rainbow universe in a more general setting. Through some explicit model constructions we find the answer is affirmative.

We organize the Letter as follows. In Section 2, we derive the modified FRW equations in the framework of rainbow universe where the cosmological evolution of probes is taken into account. Then we turn to derive the big bounce solutions to these equations with vanishing cosmological constant in Section 3. Through specifying modified dispersion relations, two sorts of big bounce solutions are demonstrated. But these models contain some unsatisfactory features at the moment when the universe passing through the big bounce. In Section 4 we show that such unsatisfactory points can be overcome by introducing an effective cosmological constant. The corresponding big bounce solutions are presented as well.

2. Modified FRW universe in rainbow gravity formalism

The modified FRW equations in rainbow gravity formalism have originally been derived in [11]. As a starting point, the conventional FRW metric is replaced by a rainbow metric parameterized by the energy ε of probes

$$ds^2 = -\frac{1}{f^2(\varepsilon)}dt^2 + \frac{a^2}{g^2(\varepsilon)}\delta_{ij}\,dx^i\,dx^j. \tag{7}$$

Here we only consider the spatially flat case with $K = 0$. This metric is defined in a general sense in that one is free to pick up arbitrary particle as a probe and for any specific measurement its energy ε can be treated as a constant which is independent of spacetime coordinates. However, in [21] rather than considering any specific measurement, this mechanism is generalized to study the semi-classical effects of particles on the background metric during a longtime process. Then the probe energy appearing in the rainbow metric is identified with one of photons or other sorts of massless particles like gravitons or inflatons which dominate the very early universe. Thus the evolution of energy ε with the cosmological time need to be taken into account, denoted as $\varepsilon(t)$. As a result the rainbow functions f and g in Eq. (7) depend on time only implicitly through the energy of particles. In [21] we take the ansatz with $g^2 = 1$. Here for our purpose f and g are chosen a priori and we will derive the modified FRW equations in a more general setting.

Directly starting from the rainbow metric in Eq. (7) and taking the cosmological evolution of ε into account, we obtain the non-vanishing components of associated connection as

$$\Gamma^0{}_{00} = -\frac{\dot f}{f}, \qquad \Gamma^i{}_{j0} = \left(\frac{\dot a}{a} - \frac{\dot g}{g}\right)\delta_{ij},$$
$$\Gamma^0{}_{ij} = \frac{f^2 a^2}{g^2}\left(\frac{\dot a}{a} - \frac{\dot g}{g}\right)\delta_{ij}. \tag{8}$$

Next the components of Ricci tensor as well as the Ricci scalar can be derived as follows

$$R_{00} = -3\left(\frac{\ddot a}{a} - \frac{\ddot g}{g} + \frac{\dot a \dot f}{af} - \frac{\dot f \dot g}{fg} - \frac{2\dot a \dot g}{ag} + \frac{2\dot g^2}{g^2}\right), \tag{9}$$

$$R_{ij} = \frac{f\dot f a\dot a + 2f^2\dot a^2 + f^2 a\ddot a}{g^2}\delta_{ij} - \left(\frac{6f^2 a\dot a\dot g + a^2 f\dot f\dot g + a^2 f^2\ddot g}{g^3} - \frac{4a^2 f^2\dot g^2}{g^4}\right)\delta_{ij}, \tag{10}$$

$$R = 6f^2\left(\frac{\ddot a}{a} + \frac{\dot a\dot f}{af} + \frac{\dot a^2}{a^2}\right) - 6f^2\left(\frac{\ddot g}{g} + \frac{\dot f\dot g}{fg} + \frac{4\dot a\dot g}{ag} - \frac{3\dot g^2}{g^2}\right). \tag{11}$$

Finally, substituting all the terms above into Eq. (6) we obtain the modified FRW equations as

$$\left(H - \frac{\dot g}{g}\right)^2 = \frac{8\pi G(\varepsilon)\rho}{3f^2} + \frac{\Lambda(\varepsilon)}{3f^2}, \tag{12}$$

$$\dot H + \frac{\dot g^2}{g^2} - \frac{\ddot g}{g} = -\frac{4\pi G(\varepsilon)(\rho + P)}{f^2} - \left(H - \frac{\dot g}{g}\right)\frac{\dot f}{f}, \tag{13}$$

where $H = \dot a/a$ is the Hubble parameter and ρ, P denote the energy density and the pressure of perfect fluids respectively.

Furthermore, the conservation law for the energy–momentum tensor reads as $\nabla(\varepsilon)^\mu T_{\mu\nu} = 0$, where the covariant derivative $\nabla(\varepsilon)^\mu$ is energy dependent as well. Plugging all the components of the connection into this equation leads to a modified conservation equation as

$$\dot\rho + 3\left(H - \frac{\dot g}{g}\right)(\rho + P) = -\frac{\dot\Lambda(\varepsilon)}{8\pi G} - \frac{\dot G}{G}\rho. \tag{14}$$

Obviously the conservation equation (14) is not independent and can be derived from Eqs. (12) and (13).

We now turn back to Eq. (7) and look at the term $a(t)/g^2$, which depends on the energy of probes through the factor g^{-2}. Obviously, at low energy limit $g^2 \to 1$, it is just the ordinary scale factor measured by observers. The cosmological singularity occurs in standard model when one tracks back to the origin of the universe and insists to take this as the physical scale factor, namely $a(t) \to 0$ as $t \to 0$ (or at some finite time). However, in rainbow gravity formalism we find that as the energy of those particles *evolving along with spacetime geometry* increase at the very early stage of the universe, the factor g^2 may be far from the unit and its effects cannot be ignored. Then any possible measurement by *those* particles is energy dependent such that the actual and physical scale factor measured by a probe depends on its energy ε through the function g^2, which is quite different from the ordinary scale factor $a(t)$. As a matter of fact, in this formalism as $a(t) \to 0$, the rainbow function g^{-2} probably becomes divergent such that the total effective scale factor probed by particles may be finite, providing a mechanism of avoiding the singularity. Therefore we define an effective scale factor $a_{eff} = a/g$ and subsequently the effective Hubble parameter as $H_{eff} = \dot{a}_{eff}/a_{eff} = H - \dot{g}/g$. They are the physical parameters probed by those matter evolving with spacetime geometry in very early universe. Through this Letter we will focus on analyzing the solutions to the effective scale factor a_{eff}.

3. The big bounce solutions with $\Lambda = 0$

Now we are going to look for big bounce solutions to the modified FRW equations (12) and (13). For simplicity through this Letter we will treat the Newton's constant G to be energy independent. Next we need to specify the rainbow functions $f(\varepsilon)$ and $g(\varepsilon)$. It has been pointed out in [21] that the effective FRW equations appearing in loop quantum cosmology [42–44] can be heuristically derived from the framework of rainbow gravity once the modified dispersion relation is properly assigned. For instance, the effective equation for $K = 0$ and $\Lambda = 0$ in loop quantum gravity has the form

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{15}$$

where $\rho_c \sim l_p^{-4}$. They may be obtained from the side of rainbow gravity if we naively set that

$$f^2 = \left(1 - l_p^4\varepsilon^4\right)^{-1}, \qquad g^2 = 1, \tag{16}$$

and $\varepsilon^4 \sim \bar{\varepsilon}^4 \sim \rho$. This identification presumably indicates a relation between the rainbow gravity formalism and the effective theory of loop quantum gravity at the semi-classical level. As a result it appears that the big bounce solutions should be obtained in rainbow universe straightforwardly. However, as pointed out in [21], this naive identification involves more subtleties to clarify from the side of doubly special relativity. The reason is that the modification of dispersion relations will provide corrections to the expectation value of statistical quantities of an ensemble as well. If a modified dispersion relation does not manifestly provide an upper bound on either the momentum or energy of a single particle, it would lead to a divergent density of states as the energy approaches to the Planck scale, which can be seen from the following definition [21, 45]

$$\begin{aligned} g(\varepsilon)\,d\varepsilon &= 2\frac{V}{h^3}4\pi p^2\,dp \\ &= \frac{8\pi V}{h^3c^3}f^3\left(1 + \varepsilon\frac{f'}{f}\right)\varepsilon^2\,d\varepsilon, \end{aligned} \tag{17}$$

where prime denotes the derivative with respect to the energy ε. To preserve the finiteness of the mean value of statistical quantities with the use of Eq. (17), it seems that some certain cutoff of the energy should be introduced by hand, which obviously sounds not satisfying. A more detailed analysis shows that this difficulty does arise when one intends to derive the big bounce solution in rainbow universe through the ansatz (16). To avoid this weak point, we propose the following two sorts of modified dispersion relations and then discuss the solutions to the corresponding modified FRW equations.

3.1. The case of $f^2 = g^2 = \frac{1}{1-l_p^4\varepsilon^4}$

The first sort of rainbow functions we would like to adopt is

$$f^2 = g^2 = \frac{1}{1 - l_p^4\varepsilon^4}. \tag{18}$$

Obviously in any case of $f^2 = g^2$, the ordinary relation $d\varepsilon = dp$ is preserved for massless particles such that all the statistical quantities will not receive corrections due to the modification of dispersion relations. In particular, the equation of state $P = \omega\rho$ and the relation $\rho \sim \bar{\varepsilon}^4$ still hold for those particles.

Next we identify the energy appearing in rainbow metric with the statistical mean value of all massless radiation particles, namely $\varepsilon \sim \bar{\varepsilon}$. This is because we are concerned with the "average" effect of radiation particles which dominate the very early universe, rather than picking up any specific particle from the radiation at random. The strategy of treating all radiation particles as an ensemble and considering their statistical effects has previously been applied to investigate the thermodynamics of black holes as well [18,19,46]. As a result, the rainbow functions of f and g can be finally expressed as

$$f^2 = g^2 = \frac{1}{1 - l_p^4\bar{\varepsilon}^4} = \frac{1}{1 - l_p^4\rho}. \tag{19}$$

Correspondingly the modified FRW equations read as

$$\frac{\dot{\rho}}{\rho} = -3H_{eff}(1+\omega), \tag{20}$$

$$H_{eff}^2 = \frac{8\pi G\rho}{3}\left(1 - l_p^4\rho\right), \tag{21}$$

which are exactly the effective FRW equations arising in loop quantum gravity. Without surprise when ω is a constant but not equal to -1, we have the big bounce solution as

$$\rho = \frac{1}{(Ct)^2 + l_p^4}, \tag{22}$$

and

$$a_{eff} = \left[(Ct)^2 + l_p^4\right]^{\frac{1}{3(1+\omega)}}, \tag{23}$$

where $C = \frac{3(1+\omega)}{2}\sqrt{\frac{8\pi G}{3}}$. It is easy to find that the energy density ρ is bounded at the big bounce of the universe, namely

$$\rho \to \frac{1}{l_p^4}, \quad \text{as } t \to 0. \tag{24}$$

Therefore, in rainbow universe the cosmological singularity can be avoided. For explicitness, we plot the evolution of the effective scale factor for two special cases with $\omega = -2/3$ and $\omega = 1/3$, as illustrated in Figs. 1 and 2 respectively.

From Eq. (23), we find there is always a big bounce for the very early universe if $\omega > -1$. However, some issues arise when one attempts to understand the behavior of the universe at the moment $t = 0$. Though both the energy density and the effective scale factor are finite at $t = 0$, the rainbow functions $f^2 = g^2 = \frac{1}{1-l_p^4\rho}$ approach to infinity as $\rho \to 1/l_p^4$ such that the time-like component of the

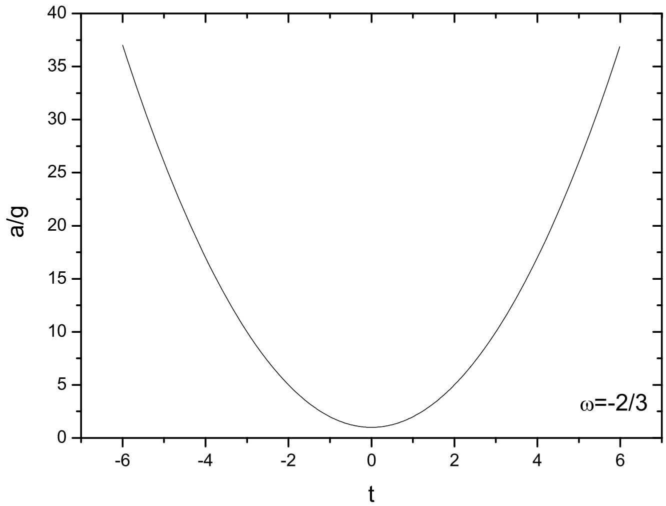

Fig. 1. The effective scale factor a_{eff} vary with the cosmological time when $f^2 = g^2 = \frac{1}{1-l_p^4\rho}$, $\omega = -\frac{2}{3}$.

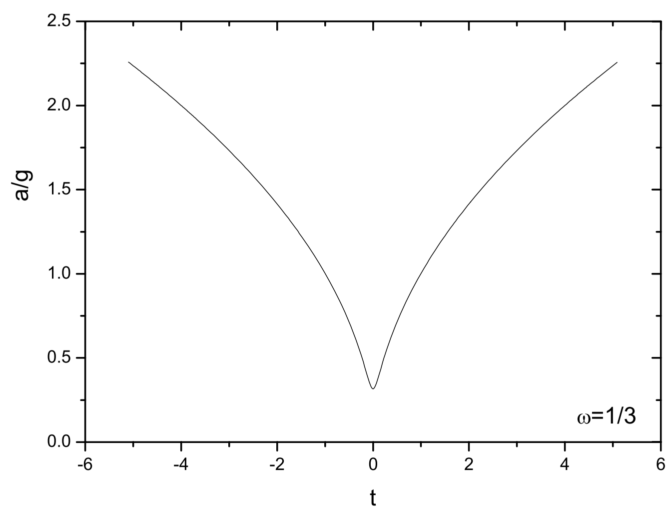

Fig. 2. The effective scale factor a_{eff} vary with the cosmological time when $f^2 = g^2 = \frac{1}{1-l_p^4\rho}$, $\omega = \frac{1}{3}$.

rainbow metric vanishes, implying the degeneracy of the metric at that time. Such difficulty is general for this sort of modified dispersion relations and cannot be avoided by inserting any adjustable parameter like $f^2 = g^2 = \frac{1}{1-\eta l_p^4\rho}$. To overcome this unsatisfactory point, we consider another sort of modified dispersion relations in next subsection.

3.2. The case of $f = g = 1 + l_p^4\bar{\varepsilon}^4$

Now we attempt to take the rainbow functions f and g as

$$f = g = 1 + l_p^4\bar{\varepsilon}^4. \tag{25}$$

Then the effective FRW equation reads as

$$H_{eff}^2 = \frac{8\pi G}{3}\frac{\rho}{(1+l_p^4\rho)^2}. \tag{26}$$

Combining this equation and the conservation equation (14) leads to the evolution equation of the energy density

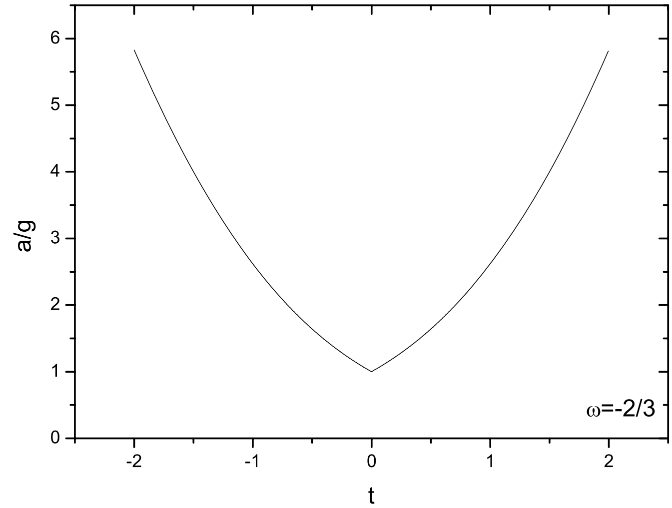

Fig. 3. The effective scale factor a_{eff} vary with the cosmological time when $f = g = 1 + l_p^4\rho$, $\omega = -\frac{2}{3}$.

$$-\frac{\dot{\rho}}{3(1+\omega)\rho} = \sqrt{\frac{8\pi G}{3}}\frac{\sqrt{\rho}}{1+l_p^4\rho}. \tag{27}$$

The solution to this equation is

$$\rho = \frac{(Ct)^2 + 2l_p^4 - \sqrt{(Ct)^4 + 4l_p^4(Ct)^2}}{2l_p^8}, \tag{28}$$

where C is a constant. Again the energy density is bounded and approaches to $\frac{1}{l_p^4}$ as $t \to 0$. Especially, in this case both rainbow functions f and g at $t = 0$ are finite such that the rainbow metric is well defined even at the moment of passing through the big bounce. Furthermore, we may obtain the effective scale factor as

$$a_{eff} = \left[\frac{(Ct)^2 + 2l_p^4 - \sqrt{(Ct)^4 + 4l_p^4(Ct)^2}}{2l_p^8}\right]^{-\frac{1}{3(1+\omega)}}. \tag{29}$$

It is also bounded and approaches to $l_p^{\frac{8}{3(1+\omega)}}$ as $t \to 0$, thus a big bounce occurs whenever $\omega \neq -1$. For instance, we illustrate its evolution with $\omega = -2/3$ in Fig. 3. From this figure, however, we see that the evolution at the moment of big bounce is *not* smooth. This comes from the fact that the solution (29) contains a square root such that $\dot{a}_{eff}$ is not continuous at $t = 0$.

In next section, we intend to demonstrate that all the unsatisfactory points appearing in this section can be overcome by introducing a non-zero cosmological constant.

4. The big bounce in the presence of the cosmological constant

In the framework of rainbow gravity, it is reasonable to expect that both the Newton's constant and the cosmological constant are dependent on the energy of probes [11]. Here we only consider the possible modification of the cosmological constant $\Lambda(\varepsilon)$. Introduce a third rainbow function h such that $\Lambda(\varepsilon) = h^2(\varepsilon)\Lambda$ where Λ is a bare cosmological constant, the modified FRW equation (12) can be written as

$$H_{eff}^2 = \frac{8\pi G\rho}{3f^2} + \frac{h^2(\varepsilon)\Lambda}{3f^2}. \tag{30}$$

To obtain the big bounce solutions and compare them with the results in previous section, we fix the functions f^2 and g^2 as

$f^2 = g^2 = \frac{1}{1-l_p^4\rho}$ and assume $h^2 = 1 + \lambda\rho$. In this setting Eq. (30) becomes

$$H_{eff}^2 = \frac{8\pi G}{3}\left[\rho\left(1 - l_p^4\rho\right) + (1+\lambda\rho)\left(1 - l_p^4\rho\right)\frac{\Lambda}{8\pi G}\right]. \tag{31}$$

Combining Eqs. (14) and (31), we have

$$\frac{\dot\rho}{\rho\sqrt{-(l_p^4 + \frac{\lambda\Lambda l_p^4}{8\pi G})\rho^2 + [1 + \frac{\Lambda}{8\pi G}(\lambda - l_p^4)]\rho + \frac{\Lambda}{8\pi G}}} \equiv -M, \tag{32}$$

where $M \equiv 3(1+\omega)\sqrt{\frac{8\pi G}{3}\frac{8\pi G}{8\pi G + \lambda\Lambda}}$. Here, we require $1 + \frac{\lambda\Lambda}{8\pi G} \neq 0$. Introducing new variables

$$x \equiv \frac{1}{\rho}, \qquad \rho_\Lambda \equiv \frac{\Lambda}{8\pi G}. \tag{33}$$

Eq. (32) can be changed into

$$\frac{\dot x}{\sqrt{\rho_\Lambda x^2 + [1 + (\lambda - l_p^4)\rho_\Lambda]x - l_p^4(1+\lambda\rho_\Lambda)}} = M. \tag{34}$$

Next we present solutions to this equation in accord with the sign of the cosmological constant Λ. Here for convenience, we define T as

$$T = \rho_\Lambda x^2 + \left[1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right]x - l_p^4(1+\lambda\rho_\Lambda). \tag{35}$$

4.1. The case of $\Lambda > 0$

When $\Lambda > 0$, we obtain the solution to Eq. (34) as

$$M\sqrt{\rho_\Lambda}t = \ln\left[2\rho_\Lambda x + \left[1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right] + 2\sqrt{\rho_\Lambda}\sqrt{T}\right] - \ln N, \tag{36}$$

where to make the big bounce occur at $t = 0$ we have introduced an integral constant $N > 0$ which is defined as

$$N^2 = \left[1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right]^2 + 4l_p^4\rho_\Lambda(1+\lambda\rho_\Lambda) > 0. \tag{37}$$

If we consider $\lambda > -\frac{1}{\rho_\Lambda}$, Eq. (37) has been satisfied. It is very useful to rewrite the solution in Eq. (36) as

$$x = \frac{1}{4\rho_\Lambda}\exp(M\sqrt{\rho_\Lambda}t + \ln N) + \frac{1}{4\rho_\Lambda}N^2\exp\left[-(M\sqrt{\rho_\Lambda}t + \ln N)\right] - \left[1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right]\frac{1}{2\rho_\Lambda}. \tag{38}$$

It is easy to show that x has the minimal value at $t = 0$,

$$x_{min} = \frac{1}{2\rho_\Lambda}\left[N - 1 - \left(\lambda - l_p^4\right)\rho_\Lambda\right] \approx \frac{l_p^4(1+\lambda\rho_\Lambda)}{1 + (\lambda - l_p^4)\rho_\Lambda}. \tag{39}$$

If the parameter λ satisfies the condition

$$\lambda > -\frac{1}{\rho_\Lambda} + l_p^4, \tag{40}$$

then we find the energy density $\rho < \frac{1}{l_p^4}$ at the big bounce. As a matter of fact, in rainbow gravity formalism it is reasonable to expect the parameter to be an order of $|\lambda| \sim l_p^4$, the condition (40) can be easily satisfied. Therefore, in the presence of a cosmological constant the rainbow function f will never diverge such that the rainbow metric is always well defined even at the big bounce. Moreover, the effective scale factor evolves as

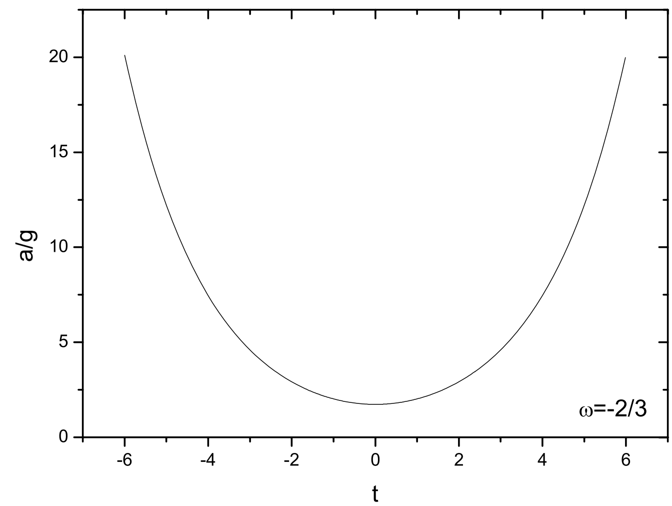

Fig. 4. The effective scale factor a_{eff} varies with the cosmological time when $\Lambda > 0$.

$$a_{eff} = \left\{\frac{1}{4\rho_\Lambda}\exp(M\sqrt{\rho_\Lambda}t + \ln N) + \frac{1}{4\rho_\Lambda}N^2\exp\left[-(M\sqrt{\rho_\Lambda}t + \ln N)\right] - \left[1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right]\frac{1}{2\rho_\Lambda}\right\}^{\frac{1+\lambda\rho_\Lambda}{3(1+\omega)}}. \tag{41}$$

We plot its evolution in Fig. 4, from which we find the big bounce occurs at $t = 0$.

4.2. The case of $\Lambda < 0$

When $\rho_\Lambda < 0$, we obtain the solution by integrating Eq. (34),

$$Mt = \sqrt{\frac{-1}{\rho_\Lambda}}\arcsin\left\{\frac{-2\rho_\Lambda x - [1 + (\lambda - l_p^4)\rho_\Lambda]}{N}\right\} + \frac{\pi}{2}\sqrt{\frac{-1}{\rho_\Lambda}}, \tag{42}$$

where $\frac{\pi}{2}\sqrt{\frac{-1}{\rho_\Lambda}}$ is introduced as an integral constant such that the big bounce shifts to $t = 0$. Similarly, we rewrite this solution as

$$x = -\frac{1}{2\rho_\Lambda}\left\{N\sin\left(M\sqrt{-\rho_\Lambda}t - \frac{\pi}{2}\right) + \left[1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right]\right\}. \tag{43}$$

Obviously x has the minimal value at $t = 0$,

$$x_{min} = \frac{1}{2\rho_\Lambda}\left[N - 1 - \left(\lambda - l_p^4\right)\rho_\Lambda\right] \approx \frac{l_p^4(1+\lambda\rho_\Lambda)}{1 + (\lambda - l_p^4)\rho_\Lambda}. \tag{44}$$

If the parameter λ is constrained as

$$-\frac{1}{\rho_\Lambda} < \lambda < -\frac{1}{\rho_\Lambda} + l_p^4, \tag{45}$$

then it is guaranteed that the energy density will never exceed the Planck energy density, namely $\rho < l_p^{-4}$ and the rainbow metric is always well defined.

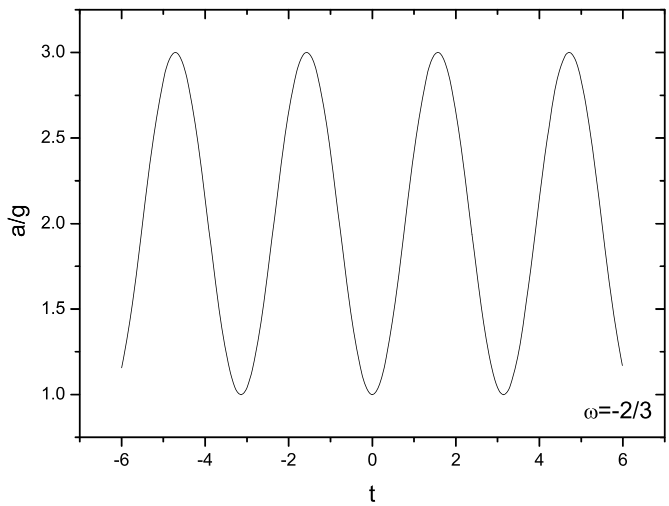

Fig. 5. The effective scale factor a_{eff} varies with the cosmological time when $\Lambda < 0$.

The solution of the effective scale factor is

$$a_{eff} = \left\{ -\frac{1}{2\rho_\Lambda} \left[N \sin\left(M\sqrt{-\rho_\Lambda} t - \frac{\pi}{2} \right) + \left(1 + \left(\lambda - l_p^4\right)\rho_\Lambda\right) \right] \right\}^{\frac{1+\lambda\rho_\Lambda}{3(1+\omega)}}. \tag{46}$$

We illustrate its evolution with the cosmological time in Fig. 5. This is an oscillation solution containing many big bounces.

Similar big bounce solutions can be found in the context of brane scenarios and effective loop quantum cosmology [37,38], where they considered a ρ^2-type modification to the Friedmann equation due to the brane effects or discreteness of quantum geometry. Here we obtain the big bounce solutions by considering the quantum gravity effects of probes. By appropriately choosing the rainbow functions f^2, g^2 and h^2 we find that ρ^2-type modification to the Friedmann equation can be achieved in the framework of rainbow universe as well. But in contrast to brane cosmology or loop quantum gravity where the modification emerges from the brane effects or quantum geometry such that the coefficients in all correction terms are uniquely fixed, here we treat the rainbow functions f^2, g^2 and h^2 as independent modification quantities. It is also interesting to notice that in loop quantum gravity the lattice regularization need to be refined in order to assure the big bounce occurring at the Planck scale [42–44]. While in rainbow gravity formalism, we find that the magnitude of the effective scale factor at the big bounce is adjusted by the parameter λ in rainbow function h which is assumed to be the order of l_p^4. We may also compare our solutions with those appearing in low energy effective theory of string theory, for instance [33], where the existence of no-singular solutions is quite relevant to the dilatonic coupling constant. In our cases the specific form of rainbow functions plays a similar role, but the presence of the cosmological constant makes it possible to obtain bouncing solutions as well as oscillatory solutions.

5. Conclusions

In this Letter we have constructed a phenomenological model for the very early universe in the framework of rainbow gravity, in which the quantum gravity effects of probes on spacetime background is taken into account at the semi-classical level. Starting from the rainbow Einstein's equation, we derived the modified FRW equations by considering the cosmological evolution of particles. Furthermore, given appropriate rainbow functions we find that the big bounce solutions to the modified FRW equations can be obtained. Notably, to obtain a well-defined rainbow metric at the moment of the big bounce, we find it seems necessary to introduce a cosmological constant which depends on the energy of probes as well. This is a quite interesting result. It implies that a universe with a positive cosmological constant more likely undergoes a big bounce at least at this phenomenological level. A complete understanding on the behavior of the universe at the big bounce probably calls for a complete quantum theory of gravity. However, our semi-classical analysis presented here has shed light on this issue and indicates that such quantum gravity effects can contribute important corrections to the classical general relativity so as to provide a more reasonable picture on the origin of the universe. The validity of semi-classical analysis has also been strengthened by recent progress in loop quantum gravity in which the exact solutions to quantum equations are remarkably well approximated by a naive extrapolation of some semi-classical effective equations [42–44].

Recently bouncing cosmology has received a lot of attentions as the astronomical observation has released the cosmological data with much higher precisions, which provide strong evidences for a nearly scale-invariant spectrum of primordial density fluctuations [47]. First of all, theoretical investigation indicates that it is possible to generate such a spectrum during the contracting phase in various bouncing models, for instance see references [48–50]. Secondly, in contrast to the slow-roll inflationary scenario in which the non-Gaussian contribution to the density fluctuation spectrum is greatly suppressed, some bouncing models require that the universe experience a contracting phase before the hot Big-Bang expansion such that a large non-Gaussianity can be predicted [51–54]. The forthcoming observations maybe distinguish and rule out various models of the origin of the universe. The non-singular bounce solutions that we have obtained in this Letter make it possible to investigate the perturbation theory of the bouncing cosmology in rainbow gravity formalism. Our investigation along this direction is under progress.

Through the Letter we only investigate some special cases in rainbow gravity with specified functions of $f(\epsilon)$, $g(\epsilon)$ and $h(\epsilon)$. However, we would like to point out that a large class of rainbow functions may lead to bouncing solutions if they could provide a ρ^m-type modification to the Friedmann equation where m is not necessarily fixed to be square as in references. Such contributions would suppress the effective Hubble parameter until it vanishes at the big bounce. Of course, we expect the further tests relevant to Lorentz symmetry would tell us which is the proper one among all the possible modified dispersion relations.

It is completely possible to extend the scheme presented here to more general cases, for instance when the spatial metric is non-flat and the Newton's constant is also energy dependent.

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